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Power series solutions of momentum and energy boundary layer equations for a power-law shear driven flow over a semi-infinite flat plate

G.E. Cossali *

Facoltà di Ingegneria-Università di Bergamo, via Marconi 6, 24044 Dalmine (BG), Italy

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Abstract

The boundary layer similarity flow past an impermeable flat plate, driven by a power law velocity profile $U = \gamma y^{\alpha}$, $y \to \infty$ is considered and power series solutions of the momentum equation, valid for all the allowed range of the parameter α, are presented. The convergence radius of the proposed solutions is estimated and a comparison with numerical solutions is reported. The boundary layer energy equation is then considered for all the wall temperature profiles that admits similarity solutions and power series solutions are given for the full range of the wall temperature profile parameter n.

Keywords: Boundary layers; Similarity solutions; Power series

1. Introduction

Convection over a semi-infinite flat plate is one of the best known problems in fluid mechanics, as its first analytic solution for the laminar case dates back to the beginning of the last century with Blasius (1908) [1] paper. Since then, the characteristics of momentum and energy transfer over a flat plate have been extensively studied, both numerically and analytically, and similarity solutions for a large variety of boundary conditions were proposed, like for stretching walls [2–5], porous media (see [6,7]), permeable surfaces [4,8], etc. Although the numerical approach allows to study more complex boundary conditions, the importance of analytical solutions is undeniable and it is witnessed by the large amount of work performed, particularly in recent years, on this subject. Recently a fully analytical solution (i.e. not relying on any approximation) of the Blasius problem has been found by Liao [9] using the homotopy analysis method. The case of adjustment of the laminar boundary layer to en exterior velocity profile of the form $U = \gamma\tilde{y}^{\alpha}$, $\tilde{y} \to \infty$ was recently investigated by Weidman et al. [10] for a large range of value of the power law parameter α and an analytical solution of the momentum equation in terms of Airy function was proposed for the case $\alpha = -1/2$. Then, Magyari et al. [11] found analytical solution for the same problem (for $\alpha = -1/2$ and $\alpha = -2/3$) with permeable wall. For the same external velocity profile, Magyari et al. [12] extended the study to the heat transfer problem and found an exact analytical solution for the energy equation in terms of Airy function for the isothermal and adiabatic impermeable wall cases (again for $\alpha = -1/2$), while in [13] an analysis of the existence of similarity solutions of energy and momentum equations was proposed. The objective of the present paper is to show the existence of power series solutions for the momentum boundary layer equation under the general case of the power-law velocity profile, thus extending the classical Blasius result for the no-shear case, and for the energy equation for all the conditions that may assure the existence of similarity solutions (see also [13]). As pointed out by Liao [14], Blasius power series solution is

* Tel.: +39 0352052309; fax: +39 035562779.
E-mail address: gianpietro.cossali@unibg.it

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Nomenclature

a	thermal diffusivity
Pr	Prandtl number
T	temperature
$\widetilde{u}$	velocity along the plate
U	free stream velocity
$\widetilde{v}$	velocity normal to the plate
Greek symbols	
Θ	non-dimensional temperature
ν	kinematic viscosity
σ	second derivative of f at $\eta = 0$: $\sigma = f''(0)$
Φ	non-dimensional temperature gradient at the fluid–solid interface
Indexes	
c	convergence
w	wall
∞	free stream

a semi-analytic one, as it needs the value of $f''(0)$ which must be given by numerical techniques, and it is only valid on a limited range of values of η (the convergence radius of the power series). The solutions presented here suffer of the same limitations, nevertheless they are fully justified by the generalisation of the Blasius result to the exterior shear flow and by the simplicity of the power series form and the easiness in calculating the coefficients. Moreover, as already pointed out by [10] and [12], this case has a variety of technical and environmental applications and the availability of analytic (or semi-analytic) solutions can help in extending the results to many applied fields.

2. Basic equations

Consider an incompressible steady laminar boundary layer flow over a semi-infinite impermeable flat plate, neglecting buoyancy and viscous dissipation and with zero pressure gradient, the basic equations describing the conservation of mass, momentum and energy in the boundary layer are given by [15,16]

$$\frac{\partial \widetilde{u}}{\partial \widetilde{x}} + \frac{\partial \widetilde{v}}{\partial \widetilde{y}} = 0 \tag{1}$$

$$\widetilde{u}\frac{\partial \widetilde{u}}{\partial \widetilde{x}} + \widetilde{v}\frac{\partial \widetilde{u}}{\partial \widetilde{y}} = \nu\frac{\partial^2 \widetilde{u}}{\partial \widetilde{y}^2} \tag{2}$$

$$\widetilde{u}\frac{\partial T}{\partial \widetilde{x}} + \widetilde{v}\frac{\partial T}{\partial \widetilde{y}} = a\frac{\partial^2 T}{\partial \widetilde{y}^2} \tag{3}$$

where $\widetilde{x}$ and $\widetilde{y}$ are the coordinates measured, respectively, along and normal to the plate. After non-dimensionalising the variables by $x = \frac{\widetilde{x}}{L}$; $y = \frac{\widetilde{y}}{L}$; $u = \frac{\widetilde{u}L}{\nu}$; $v = \frac{\widetilde{v}L}{\nu}$, and defining the similarity coordinate $\eta = yx^{m-1}$, the equations reduce to

$$f''' + mff'' - f'f'(2m-1) = 0 \tag{4}$$

$$Prxf'T_x - mPrfT' - T'' = 0 \tag{5}$$

where $f(\eta) = \psi x^{-m}$, and ψ is the non-dimensional stream function. These equations must be solved subject to the non-slip boundary conditions on the wall and, following [10] and [12], we will consider the general case of an exterior power law velocity profile, then the boundary conditions for the momentum equation become

$$f'(0) = 0;\quad f(0) = 0;\quad f'(y = \infty) = \beta\eta^{\alpha}$$

and the last one imposes $m(\alpha) = \frac{(\alpha+1)}{(\alpha+2)}$, (the classical case with no shear ($\alpha = 0$) is obtained for $m = \frac{1}{2}$). Similarity solutions of the energy equation (5) can be found by choosing appropriate boundary conditions. The transformation:

$$T = P(x)\Theta(\eta, x) + T_\infty;\quad \Theta(\eta, x) = \frac{T(x,\eta) - T_\infty}{T(x,0) - T_\infty};$$

$$P(x) = T(x,0) - T_\infty$$

assures the existence of similarity solutions (i.e. Θ explicitly independent of x) of Eq. (5) when $P(x) = T(x,0) - T_\infty = Ax^n$, provided $A \neq 0$ [13], yielding the non-dimensional form of the energy equation

$$\Theta''_{m,n} + m(\alpha)Prf\Theta'_{m,n} - nPrf'\Theta_{m,n} = 0 \tag{6}$$

with boundary conditions: $\Theta_{m,n}(x,0) = 1$; $\Theta_{m,n}(x,\infty) = 0$. As noticed by Weidman et al. [10] the characteristic length L can be arbitrarily chosen, and the choice $L = (\frac{\nu}{\gamma})^{\frac{1}{1+\alpha}} \Rightarrow \beta = 1$ simplifies the problem. Due to this degree of freedom, in the remainder of the paper the value of β will be set equal to 1 (and $L = (\frac{\nu}{\gamma})^{\frac{1}{1+\alpha}}$) without loss of generality.

3. Power series solutions of the momentum equation

The solution f of the momentum equation (4) has the following property: the only derivatives of f different from zero at the origin are: $f^{(3k+2)}$(the notation $f^{(k)} = \frac{d^k f}{d\eta^k}$ will be used throughout the paper), in other words

$$f^{(3k)}(0) = 0;\quad f^{(3k+1)}(0) = 0;\quad f^{(3k+2)}(0) \neq 0; \tag{7}$$

for any integer $k \geqslant 0$. The statement (7) is true for $k = 0$, in fact

$$f(0) = 0;\quad f^{(1)}(0) = 0;\quad f^{(2)}(0) \neq 0. \tag{8}$$

Taking the r-derivative of both sides of Eq. (4), applying Leibniz rule and using (8)

$$f^{(3+r)} = \sum_{k=2}^{r} p(r,k,m) f^{(k)} f^{(r-k+2)}$$

with $p(r,k,m) = \left[(2m-1)\begin{pmatrix} r \\ k-1 \end{pmatrix} - m\begin{pmatrix} r \\ k \end{pmatrix}\right]$. Suppose now that the statement (7) is true at least for $0 \leqslant s \leqslant h$ for a given $h > 0$, i.e.

$$f^{(3s)}(0) = 0; \quad f^{(3s+1)}(0) = 0; \quad f^{(3s+2)}(0) \neq 0;$$

for $0 \leqslant s \leqslant h$, then

$$f^{(3h+3)}(0) = \sum_{s=0}^{h-1} p(3h, 3s+2, m) f^{(3s+2)}(0) f^{(3[h-s])}(0) = 0$$

$$f^{(3h+4)}(0) = \sum_{k=0}^{h-1} p(3h+1, 3s+2, m) f^{(3s+2)}(0) f^{(3[h-s]+1)}(0) = 0$$

$$f^{(3h+5)}(0) = p(3h+2, 2, m) f^{(2)}(0) f^{(3h+2)}(0) + \sum_{k=3}^{3h+2} p(3h+2, k, m) f^{(k)}(0) f^{(3h-k+4)}(0) \neq 0$$

showing that the statement holds also for $s = h+1$, thus the statement (7) is proven by induction. Setting $G^{(s)} = f^{(3s+2)}$ we can write

$$G^{(0)} = f^{(2)}(0) = \sigma; \quad G^{(z+1)} = \sum_{s=0}^{z} q_m(z,s) G^{(s)} G^{(z-s)} \tag{9}$$

with $q_m(z,s) = \left[(2m-1)\begin{pmatrix} 3z+2 \\ 3s+1 \end{pmatrix} - m\begin{pmatrix} 3z+2 \\ 3s+2 \end{pmatrix}\right]$. It is also easy to show that: $G^{(s)} = M_m^{(s)}\sigma^{s+1}$ and substituting into Eq. (9) the following recursive definition of $M_m^{(s)}$ is found:

$$M_m^{(s+1)} = \sum_{k=0}^{s} q_m(s,k) M_m^{(k)} M_m^{(s-k)}$$

with $M_m^{(0)} = 1$, explicitly: $M_m^{(1)} = \{3m-2\}; M_m^{(2)} = \{9m-10\}\{3m-2\}; M_m^{(3)} = \{279m^2 - 552m + 300\}\{3m-2\}$ etc. The Taylor expansion of f and f' are

$$f(\eta) = \sum_{s=0} \frac{M_m^{(s)}\sigma^{s+1}}{(3s+2)!}\eta^{3s+2};$$

$$f'(\eta) = \sum_{s=0} \frac{M_m^{(s)}\sigma^{s+1}}{(3s+1)!}\eta^{3s+1} \tag{10}$$

Fig. 1 reports the approximations obtained by the partial sums (thin lines) compared to the numerical solution (thick line) obtained by a 4th-order Runge–Kutta method, following a procedure similar to that reported by [10]. The convergence radius of the series (10) can be found by applying the ratio test, obtaining:

$$\eta_c = 3\sigma^{-1/3}\lim_{s\to\infty}\left|\frac{s^3 M_m^{(s)}}{M_m^{(s+1)}}\right|^{1/3}$$

The first 55 terms of the sequence $s_n = \left|\frac{n^3 M_m^{(n)}}{M_m^{(n+1)}}\right|^{1/3}$ were evaluated and an extrapolation method was applied for accelerating the convergence, after noticing that $s_{n+1} - s_n \simeq \frac{A}{n^p}$, and the results are reported in Table 1 and Fig. 2.

It is of some interest to observe that the further transformation $Z = \sigma^{-1/3} f$, $X = \sigma^{1/3}\eta$ yields a solution independent of σ

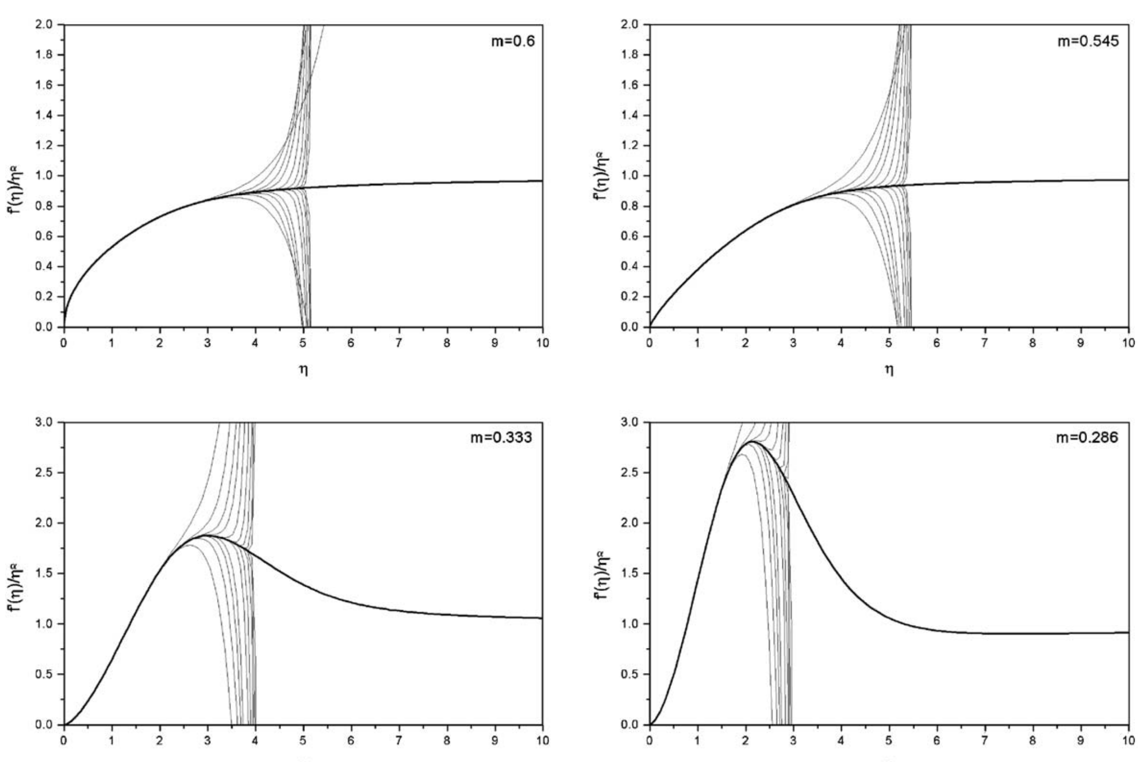

Fig. 1. Power series solutions of the momentum equation for different values of the parameter α. The truncated power series are compared to the numerical solutions (thicker line).

Table 1
Estimated convergence radius for series (10) (column 3) and (11) (column 4) and values of σ estimated from $\sigma = \left(\frac{X^\alpha}{Z_x(\infty)}\right)^{\frac{3}{2+\alpha}}$ (column 5) and numerically evaluated (column 6)

α	m	η_c	X_c	$\sigma_{\text{est.}}$	$\sigma_{\text{num.}}$
0.5	0.6	5.283	4.282	0.522	0.5325
0.2	0.545	5.603	4.069	0.386	0.3831
0	0.5	5.691	4.069	0.335	0.33206
−0.1	0.4737	5.646	3.940	0.318	0.3236
−0.2	0.444	5.511	3.876	–	–
−0.3333	0.4	5.125	3.812	–	–
−0.5	0.333	4.131	3.724	–	–
−0.6	0.2857	3.063	3.537	–	–

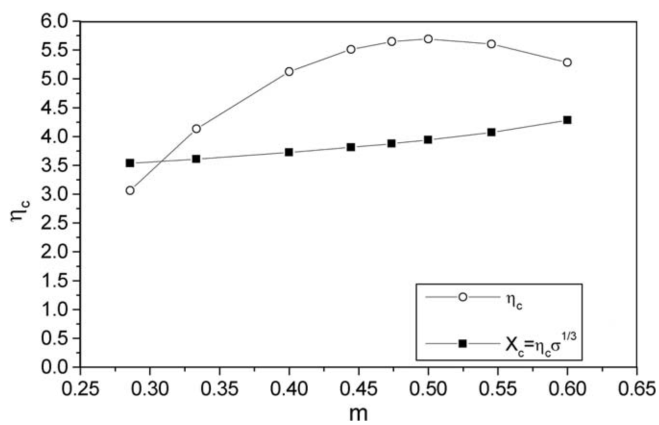

Fig. 2. Convergence radius of the momentum equation power series solutions for different values of the parameter $m = \frac{\alpha+1}{\alpha+2}$.

$$Z(X) = \sum_{s=0} \frac{M_m^{(s)}}{(3s+2)!} X^{3s+2} \tag{11}$$

and the convergence radius X_c is reported in Table 1 and Fig. 2. Now $Z_{xx}(0) = 1$; and $\frac{Z_x(\infty)}{X^\alpha} \to \sigma^{-\frac{2+\alpha}{3}}$, and if the convergence radius is large enough to allow a safe estimation of the asymptotic value $Z_x(\infty)$, the value of σ can be approximated by $\sigma = (\frac{X^\alpha}{Z_x(\infty)})^{\frac{3}{2+\alpha}}$. Table 1 shows the results for the narrow range of values of α (from -0.1 to 0.5) for which the approximation (obtained by the power series solution truncated to the 55th term) is lower than 2%. It is worth to remark that this method is analogous to that used by Blasius to give the first roughest approximation to σ for $\alpha = 0$ [1].

4. Power series solutions of the energy equation

Consider the solution $\Theta_{m,n}$ of the energy equation (6) the following result holds: the derivatives of $\Theta_{m,n}$ at the origin satisfy the relations

$$\begin{aligned} &\Theta_{m,n}^{(3s)}(0) = H_{m,n}^{(s-1)}(Pr)\sigma^s \\ &\Theta_{m,n}^{(3s+1)}(0) = K_{m,n}^{(s-1)}(Pr)\sigma^s \Phi_{m,n} \\ &\Theta_{m,n}^{(3s+2)}(0) = 0 \end{aligned} \tag{12}$$

for any $s \geqslant 0$ with $\Phi = -\left(\frac{\partial\Theta}{\partial\eta}\right)_{\eta=0}$. The statement is true for $s = 0$, in fact, considering the energy equation (6), from the boundary conditions at $\eta = 0$ it is easy to see that: $\Theta(0) = 1$, $\Theta^{(1)} = -\Phi, \Theta^{(2)}(0) = 0$, for any m and n , yielding the values $H_{m,n}^{(-1)} = 1$ and $K_{m,n}^{(-1)} = -1$. Suppose now that (12) holds for any $s \leqslant k-1$ for a given value of $k > 1$. Taking the h-derivative of both side of Eq. (6), applying Leibniz rule and considering Eq. (7)

$$\Theta_{m,n}^{(h+2)}(0) = -mPr \sum_{p=0}^{\frac{h-2}{3}} \binom{h}{3p+2} G^{(p)} \Theta_{m,n}^{(h-3p-1)} + nPr \sum_{p=0}^{\frac{h-1}{3}} \binom{h}{3p+1} G^{(p)} \Theta_{m,n}^{(h-3p-1)}$$

so that

$$\begin{aligned} \Theta_{m,n}^{(3k+1)}(0) &= Pr\left\{\sum_{p=0}^{k-1}\left[-m\binom{3k-1}{3p+2} + n\binom{3k-1}{3p+1}\right]\right\} \\ &\quad\times M_m^{(p)} K_{m,n}^{(k-p-2)}(Pr)\sigma^k \Phi_{m,n} \\ \Theta_{m,n}^{(3k+2)}(0) &= Pr\sum_{p=0}^{k-1}\left[-m\binom{3k}{3p+2} + n\binom{3k}{3p+1}\right] \\ &\quad\times G^{(p)}\Theta^{(3(k-p-1)+2)}(0) = 0 \\ \Theta_{m,n}^{(3k+3)}(0) &= Pr\left[\sum_{p=0}^{k-1}\left[-m\binom{3k+1}{3p+2} + n\binom{3k+1}{3p+1}\right]\right. \\ &\quad\left.\times H_{m,n}^{(k-p-1)}(Pr)M_m^{(p)} + nM_m^{(k)}\right]\sigma^{k+1} \end{aligned}$$

and Eq. (12) hold also for $s = k$, with

$$H_{m,n}^{(h)} = Pr\left[\sum_{p=0}^{h-1}\left[-m\binom{3h+1}{3p+2} + n\binom{3h+1}{3p+1}\right] \times H_{m,n}^{(h-p-1)}(Pr)M_m^{(p)} + nM_m^{(h)}\right] \tag{13}$$

$$K_{m,n}^{(h)} = Pr\sum_{p=0}^{h}\left[-m\binom{3h+2}{3p+2} + n\binom{3h+2}{3p+1}\right] M_m^{(p)} K_{m,n}^{(h-p-1)}(Pr) \tag{14}$$

the statement is therefore proven by induction. The recursive formulas (13,14) with the conditions: $H_{m,n}^{(-1)} = 1$, $K_{m,n}^{(-1)} = -1$ allow to evaluate all the derivatives. The explicit calculation of the coefficients $H_{m,n}^{(s)}(Pr)$ and $K_{m,n}^{(s)}(Pr)$ shows (and it is easy to demonstrate that) such coefficients can be expressed as polynomials in Pr, i.e.,

$$H_{m,n}^{(-1)} = 1 \qquad H_{m,n}^{(s)} = nPr\sum_{k=0}^{s} C_k(m,n)Pr^k \quad \text{for } s \geqslant 0$$

$$K_{m,n}^{(-1)} = -1 \quad K_{m,n}^{(s)} = Pr\sum_{k=0}^{s} D_k(m,n)Pr^k \quad \text{for } s \geqslant 0$$

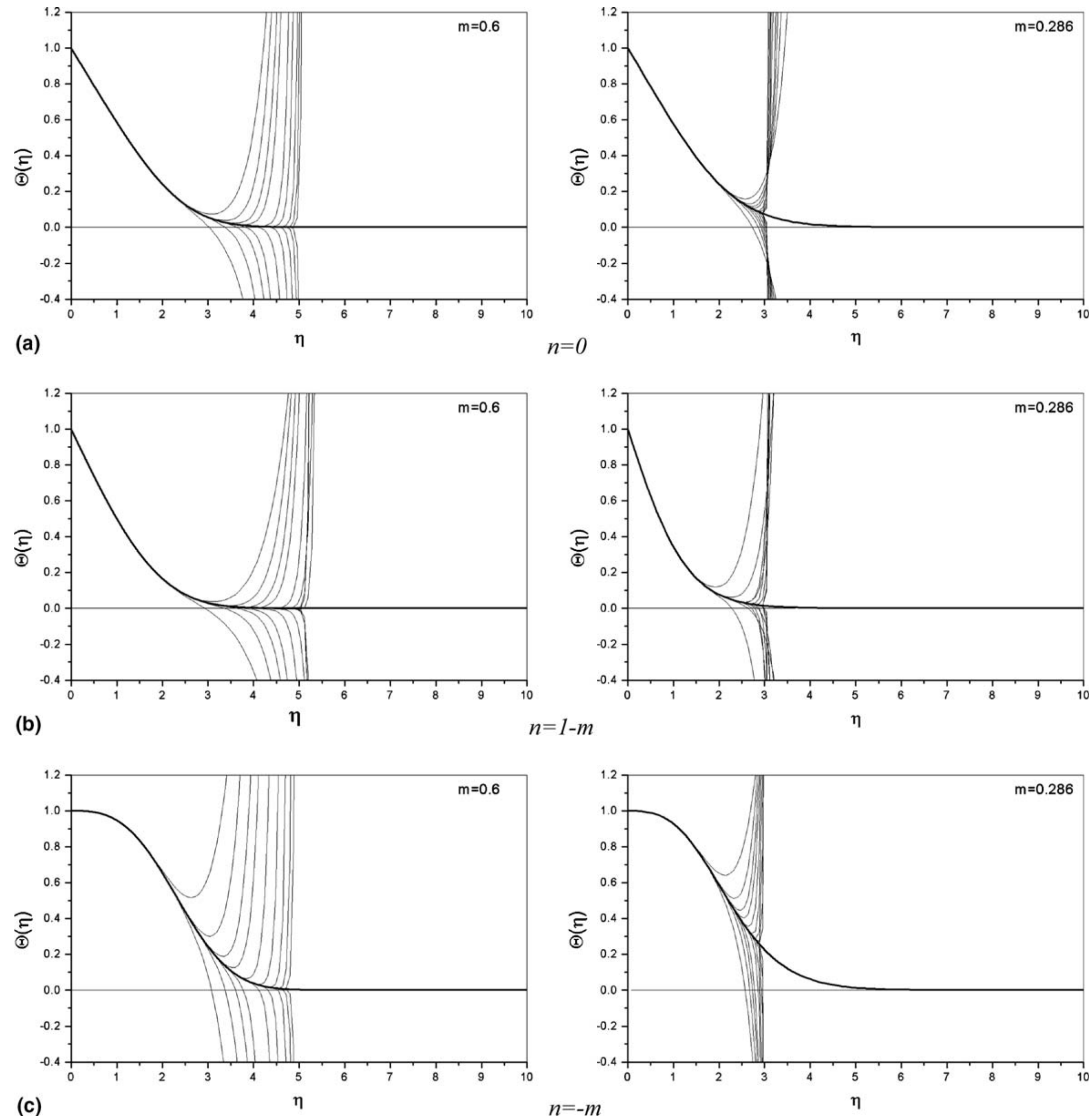

Fig. 3. Power series solutions of the energy equation for $\alpha = 0.5$. and $\alpha = -0.6$ and for the three thermal boundary conditions: (a) isothermal wall; (b) uniform heat flux; (c) adiabatic wall. The thick line is the numerical solution.

The Taylor expansion of the function $\Theta_{m,n}$ can now be written as

$$\Theta_{m,n} = 1 + \sum_{j=1}^{\infty} \frac{\Theta_{m,n}^{(j)}}{j!}\eta^j = \sum_{s=0}^{\infty} \frac{H_{m,n}^{(s-1)}(Pr)\sigma^s}{3s!}\eta^{3s+3} + \Phi_{m,n}\sum_{s=0}^{\infty} \frac{K_{m,n}^{(s-1)}(Pr)\sigma^s}{(3s+1)!}\eta^{3s+1} \quad (15)$$

Fig. 3 reports the approximation obtained by the partial sums (thin lines) compared to the numerical solution (thick line) obtained again by a 4th-order Runge–Kutta shooting method (see also [13]).

5. Some special cases

Consider now some special cases for the momentum equation solutions. For $m = 1/2$ the well known Blasius result is recovered, as $q_{1/2}(z,s) = -\frac{1}{2}\binom{3z+2}{3s+2}$ [1]. For $m = 2/3$ ($\alpha = 1$) all the coefficients $M_{2/3}^{(j)} = 0$, for $j > 0$, then $f'(\eta) = \sigma\eta$ finding the classical Couette solution ($\sigma = 1$). For $m = 1/3$, Weidman et al. ([10]) gave an analytical solution in terms of Airy functions;

$$f = (24)^{1/3}\frac{R^{(1)}(3^{-2/3}\eta)}{R(3^{-2/3}\eta)} \quad (16)$$

with $R(z) = \sqrt{3}Ai(z) + B(z)$, and $R^{(k)} = \frac{d^k R}{dz^k}$. The function $R(z)$ satisfies the differential equation [17]

$$R^{(2)} = zR \quad (17)$$

and $R(0) = R_0 = \frac{2}{3^{1/6}\Gamma(\frac{2}{3})}$, $R^{(1)}(0) = 0$, $R^{(2)}(0) = 0$. Evaluating the higher derivatives of $R(z)$ from Eq. (17), it is easy to show that the only derivative $R^{(k)}$ different from zero at the origin are those for which $k = 3s$ and that

$$R^{(3(s+1))}(0) = (3s+1)R^{(3s)}(0) = \prod_{k=0}^{s}(3k+1)R_0 = a_{s+1}R_0$$

where explicitly $a_j = \{1, 1, 4, 28, 280, \dots\}$ for $j = 0, 1, \dots$. Moreover, rewriting Eq. (16) as $R_z = (24)^{-1/3} fR$ and evaluating the higher order derivatives, remembering that $G^{(s)} = M_m^{(s)}\sigma^{s+1}$, we obtain the following identity:

$$R^{3(h+1)}(0) = \sum_{p=0}^{h} \frac{3h+2}{3p+2} 6^p M_{1/3}^{(p)} R^{3(h-p)}(0) \tag{18}$$

It is interesting to notice that comparing (18) to Eq. (13), setting $n = -m = -1/3$ and $Pr = -1/2$ we also obtain

$$\frac{R^{(3s)}(0)}{R_0} = H_{1/3,-1/3}^{(s-1)}\left(-\frac{1}{2}\right)6^s \tag{19}$$

and this is fully consistent with the result obtained by [12] for the adiabatic case, where $\Theta_{1/3,-1/3} = \left[\frac{R(z)}{R(0)}\right]^{-2Pr}$ and setting $Pr = -1/2$ Eq. (19) is re-obtained after expanding $R(z)$ in power series (clearly, the case $Pr = -1/2$ has no physical meaning but sets an interesting relationship between the analytical solution and the power series one). It should anyway be stressed that the power series solution is only valid for $\eta < \eta_c \approx 4.13$, whereas the analytic solution of Weidman et al. [10] (and Magyari et al. [12] for the energy equation) holds for every η. Considering the energy equation, for $n = 0$ the isothermal case is retrieved, but in this case it is easy to see that

$$H_{m,0}^{(h)} = \begin{cases} 1 & \text{for } h = -1 \\ 0 & \text{for } h \geqslant 0 \end{cases} \quad K_{m,0}^{(h)} = -mPr\sum_{p=0}^{h}\binom{3h+2}{3p+2} M_m^{(p)} K_{m,0}^{(h-p-1)}(Pr)$$

then

$$\Theta_{m,0} = 1 + \Phi\sum_{p=0}^{\infty}\frac{K_{m,0}^{(p-1)}(Pr)\sigma^p}{(3s+1)!}\eta^{3p+1}$$
$$= 1 - \Phi\eta + \Phi\frac{Prm\sigma}{4!}\eta^4 + \Phi\frac{[-10mPr + (3m-2)]Prm\sigma^2}{7!}\eta^7 + \cdots$$

For $m = 1/2$ and $Pr = 1$, the energy and momentum equation solutions are related by: $\Theta_{1/2,0} = 1 - f'$, then $M_{1/2}^{(s)} = -K_{1/2,0}^{(s-1)}$. On integrating Eq. (6) we obtain after a partial integration and taking into account the boundary conditions

$$\Phi_{n,m}(Pr) = -mPr\beta\lim_{\eta\to\infty}\eta^{\alpha}\Theta_{n,m} + (m+n)Pr\int_0^{\infty} f_m'\Theta_{n,m}\,\mathrm{d}\eta \tag{20}$$

Conjecturing (as in [13]) that $\Theta^{(n,m)}$ goes to zero faster than $\eta^{-\alpha}$ when $\eta \to \infty$ (for $m = 1/3$ this was proven by Magyari et al. [12]), then Eq. (20) becomes

$$\Phi_{n,m}(Pr) = (m+n)Pr\int_0^{\infty} f_m'\Theta_{n,m}\,\mathrm{d}\eta \tag{21}$$

and the adiabatic case is found for $n = -m$; Eq. (6) then reduces to $\Theta_{m,-m}'' = -mPr(f\Theta_{m,-m})'$ and the equation $f = -\frac{1}{mPr}\frac{\Theta_{m,-m}'}{\Theta_{m,-m}}$ which holds for any m, is the differential version of (14). It is also easy to see that, for any m, the isothermal and adiabatic solutions are related by the equation $\Theta_{m,-m} = \Phi^{-1}\Theta_{m,0}'$ and in fact:

$$H_{m,-m}^{(h)} = -Prm\sum_{p=0}^{h}\left[\binom{3h+2}{3p+2}\right]H_{m,n}^{(h-p-1)}(Pr)M_m^{(p)} \tag{22}$$

$$K_{m,0}^{(h)} = -Prm\sum_{p=0}^{h}\left[\binom{3h+2}{3p+2}\right]K_{m,n}^{(h-p-1)}(Pr)M_m^{(p)} \tag{23}$$

then $H_{m,-m}^{(h)} = -K_{m,0}^{(h)}$.

6. Conclusions

The boundary layer momentum and energy equations in the forced convection flow past an impermeable semi-infinite flat plate in outer shear flow were considered. Power series solutions of the momentum equation valid for all values of the shear parameter $\alpha = \frac{2m-1}{1-m}$ have been found and a recursive formula to evaluate the power series coefficients has been proposed. The convergence radius of the power series solution was estimated for a range of values of α. Power series solutions of the energy equation have also been found for all the conditions assuring the existence of similarity solutions and again recursive formulas to evaluate the power series coefficients have been proposed. Some special cases were analysed and connections between power series coefficients for different cases (like isothermal and adiabatic walls) were pointed out. Finally, the analytic solution allows for a narrow range of values of the parameter α (-0.1 to $+0.5$) a rough estimation (error lower than 2%) of the important parameter $f''(0)$.

References

[1] H. Blasius, Grenzschichten in Flussigkeiten mit kleiner Reibung, Z. Math Phys. 56 (1908) 1–37 (Translated in English as: The boundary layer in fluids with little friction. NACA TM 1256, (1956)).
[2] L. Crane, Flow past a stretching plate, Z. Angew. Mat. Phys. 21 (1970) 645–647.
[3] W.H.H. Banks, Similarity solutions of the boundary layer equations for a stretching wall, J. Mech.Theor. Appl. 2 (1983) 375–392.
[4] E. Magyari, B. Keller, Exact solutions for self-similar boundary-layer flows induced by permeable stretching walls, Eur. J. Mech. B – Fluids 19 (2000) 109–122.
[5] H.K. Kuiken, On boundary layers in fluid mechanics that decay algebraically along stretches of walls that are not infinitesimally small, IMA J. Appl. Math. 27 (1981) 387–405.
[6] D.B. Ingham, I. Pop, Transport Phenomena in Porous Media, Pergamon, Oxford, 2002.
[7] D.A. Nield, A. Bejan, Convection in Porous Media, Springer, New York, 1999.
[8] E. Magyari, I. Pop, B. Keller, New analytical solutions of well-known boundary value problem in fluid mechanics, Fluid Dynamics Res. 33 (2003) 313–317.
[9] S.-J. Liao, An explicit, totally analytic, solution for Blasius viscous flow problems, Int. J. Non-Lin. Mech. 34 (1999) 758–778.
[10] P.D. Weidman, D.G. Kubitschek, S.N. Brown, Boundary layer similarity flow driven by power-law shear, Acta Mechanica 120 (1997) 199–215.

[11] E. Magyari, B. Keller, I. Pop, Boundary layer similarity flows driven by a power-law shear over a permeable plane surface, Acta Mechanica 163 (2003) 139–146.
[12] E. Magyari, B. Keller, I. Pop, Heat transfer characteristics of a boundary-layer flow driven by a power-law shear over a semi-infinite plate, Int. J. Heat Mass Transfer 47 (2004) 31–34.
[13] G.E. Cossali, Similarity solutions of energy and momentum boundary layer equations for a power-law shear driven flow over a semi-infinite flat plate, Eur. J. Mech. B – Fluids 25 (2006) 18–32.
[14] S.-J. Liao, A kind of approximate solution technique which does not depend upon small parameters-II. An application in fluid mechanics, Int. J. Non-Linear Mech. 32 (1997) 815–822.
[15] A. Bejan, Convection Heat Transfer, J. Wiley & Sons, NY, 1995.
[16] W.M. Kays, M.E. Crawford, Convective Heat and Mass Transfer, third ed., McGrawHill, N.Y, 1993.
[17] M. Abramowitz, I.A. Stegun, Handbook of Mathematical Functions, Dover, New York, 1966.